The Derivative Proof Google Docs

By Kasen Fox

Thank you to Andrew Coleman and William Upton

Derivative of a Constant

$$\frac{d}{dx}[c] = \lim_{h \to 0} \frac{c - c}{h}$$
$$\frac{d}{dx}[c] = \lim_{h \to 0} 0$$
$$\frac{d}{dx}[c] = 0$$

Scalar Rule

$$\frac{d}{dx}[cf(x)] = \lim_{h \to 0} \frac{cf(x+h) - cf(x)}{h}$$
$$\frac{d}{dx}[cf(x)] = c \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$\frac{d}{dx}[cf(x)] = cf'(x)$$

Addition Rule

$$\frac{d}{dx}[f(x) + g(x)] = \lim_{h \to 0} \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h}$$
$$\frac{d}{dx}[f(x) + g(x)] = \lim_{h \to 0} \frac{f(x+h) - f(x) + g(x+h) - g(x)}{h}$$

$$\frac{d}{dx}[f(x)+g(x)] = \lim_{h\to0}\frac{f(x+h)-f(x)}{h} + \lim_{h\to0}\frac{g(x+h)-g(x)}{h}$$
$$\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x)$$

Subtraction Rule

$$\frac{d}{dx}[f(x)-g(x)] = \lim_{h\to0}\frac{f(x+h)-g(x+h)-f(x)+g(x)}{h}$$
$$\frac{d}{dx}[f(x)-g(x)] = \lim_{h\to0}\frac{f(x+h)-f(x)-(g(x+h)-g(x))}{h}$$
$$\frac{d}{dx}[f(x)-g(x)] = \lim_{h\to0}\frac{f(x+h)-f(x)}{h} - \lim_{h\to0}\frac{g(x+h)-g(x)}{h}$$
$$\frac{d}{dx}[f(x)-g(x)] = f'(x)-g'(x)$$

Product Rule

$$\frac{d}{dx}[f(x)g(x)] = \lim_{h\to0}\frac{f(x+h)g(x+h)-f(x)g(x)}{h}$$
$$\frac{d}{dx}[f(x)g(x)] = \lim_{h\to0}\frac{f(x+h)g(x+h)-f(x+h)g(x)+f(x+h)g(x)-f(x)g(x)}{h}$$
$$\frac{d}{dx}[f(x)g(x)] = \lim_{h\to0}\frac{f(x+h)g(x+h)-f(x+h)g(x)}{h} + \lim_{h\to0}\frac{f(x+h)g(x)-f(x)g(x)}{h}$$
$$\frac{d}{dx}[f(x)g(x)] = \lim_{h\to0}f(x+h)\lim_{h\to0}\frac{g(x+h)-g(x)}{h} + \lim_{h\to0}g(x)\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$
$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x)+g(x)f'(x)$$

Quotient Rule

$$\frac{d}{dx}[\frac{f(x)}{g(x)}] = \lim_{h\to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$$

$$\frac{d}{dx}[\frac{f(x)}{g(x)}] = \lim_{h\to 0} \frac{1}{h} \frac{f(x+h)g(x) - g(x+h)f(x)}{g(x+h)g(x)}$$

$$\frac{d}{dx}[\frac{f(x)}{g(x)}] = \lim_{h\to 0} \frac{1}{g(x+h)g(x)} \frac{f(x+h)g(x) - f(x)g(x) + f(x)g(x) - g(x+h)f(x)}{h}$$

$$\frac{d}{dx}[\frac{f(x)}{g(x)}] = \lim_{h\to 0} \frac{1}{g(x+h)g(x)} (\frac{f(x+h)g(x) - f(x)g(x)}{h} + \frac{f(x)g(x) - g(x+h)f(x)}{h})$$

$$\frac{d}{dx}[\frac{f(x)}{g(x)}] = \lim_{h\to 0} \frac{1}{g(x+h)g(x)} (g(x)\frac{f(x+h) - f(x)}{h} - f(x)\frac{g(x+h) - g(x)}{h})$$

$$\frac{d}{dx}[\frac{f(x)}{g(x)}] = \lim_{h\to 0} \frac{1}{g(x+h)g(x)} \lim_{h\to 0} g(x) \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} - \lim_{h\to 0} f(x) \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$

$$\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{1}{g(x)g(x)} (f'(x)g(x) - g'(x)f(x))$$

$$\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

Derivative of y=x

$$\frac{d}{dx}[x] = \lim_{h\to 0} \frac{x + h - x}{h}$$

$$\frac{d}{dx}[x] = \lim_{h\to 0} \frac{h}{h}$$

$$\frac{d}{dx}[x] = \lim_{h\to 0} 1$$

$$\frac{d}{dx}[x] = 1$$

Derivative of y=x^2

$$\frac{d}{dx}[x^2] = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$$
$$\frac{d}{dx}[x^2] = \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$
$$\frac{d}{dx}[x^2] = 2x$$

Derivative of x^n

$$\frac{d}{dx}[x^n] = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h}$$
$$\frac{d}{dx}[x^n] = \lim_{h\to 0} \frac{x^n + \binom{n}{1}x^{n-1}h + \binom{n}{2}x^{n-2}h^2 + \cdots + \binom{n}{n}x^0h^n - x^n}{h}$$
$$\frac{d}{dx}[x^n] = \lim_{h\to 0} \binom{n}{1}x^{n-1} + \binom{n}{2}x^{n-2}h + \cdots + \binom{n}{n}h^{n-1}$$
$$\frac{d}{dx}[x^n] = \binom{n}{1}x^{n-1}$$
$$\frac{d}{dx}[x^n] = \frac{n!}{1!(n-1)!}x^{n-1}$$
$$\frac{d}{dx}[x^n] = nx^{n-1}$$

Derivative of 1/x

$$\frac{d}{dx}[\frac{1}{x}] = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$\frac{d}{dx}[\frac{1}{x}] = \lim_{h\to 0} - \frac{\frac{h}{x(x+h)}}{h}$$
$$\frac{d}{dx}[\frac{1}{x}] = \lim_{h\to 0} - \frac{1}{x(x+h)}$$
$$\frac{d}{dx}[\frac{1}{x}] = - \frac{1}{x(x)}$$
$$\frac{d}{dx}[\frac{1}{x}] = - \frac{1}{x^2}$$

Derivative of $\sqrt{x}$

$$\frac{d}{dx}[\sqrt{x}] = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$
$$\frac{d}{dx}[\sqrt{x}] = \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$$
$$\frac{d}{dx}[\sqrt{x}] = \lim_{h\to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$
$$\frac{d}{dx}[\sqrt{x}] = \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$
$$\frac{d}{dx}[\sqrt{x}] = \frac{1}{\sqrt{x} + \sqrt{x}}$$
$$\frac{d}{dx}[\sqrt{x}] = \frac{1}{2\sqrt{x}}$$

Derivative of $1/x^n$

$$\frac{d}{dx}[\frac{1}{x^n}] = \frac{d}{dx}[x^{-n}]$$
$$\frac{d}{dx}[\frac{1}{x^n}] = -nx^{-(n+1)}$$
$$\frac{d}{dx}[\frac{1}{x^n}] = -\frac{n}{x^{n+1}}$$

Limit of sin(x)/x

$$\frac{1}{2}\tan(x) \geq \frac{1}{2}x \geq \frac{1}{2}\sin(x)$$
$$\cos(x) \leq \frac{\sin(x)}{x} \leq 1$$
$$\lim_{x\to 0}\cos(x) \leq \lim_{x\to 0}\frac{\sin(x)}{x} \leq \lim_{x\to 0} 1$$
$$1 \leq \lim_{x\to 0}\frac{\sin(x)}{x} \leq 1$$
$$\lim_{x\to 0}\frac{\sin(x)}{x} = 1$$

Limit of 1-cos(x)/x

$$\lim_{x\to 0}\frac{1-\cos(x)}{x}$$
$$\lim_{x\to 0}\frac{1-\cos^2(x)}{x(1+\cos(x))}$$
$$\lim_{x\to 0}\frac{\sin^2(x)}{x(1+\cos(x))}$$

$$\lim_{x\to 0} \sin(x)\frac{\sin(x)}{x}\frac{1}{1+\cos(x)}$$

$$\lim_{x\to 0} \sin(x) \lim_{x\to 0} \frac{\sin(x)}{x} \lim_{x\to 0} \frac{1}{1+\cos(x)}$$

$$(0)(1)(\frac{1}{2})$$

$$\lim_{x\to 0} \frac{1-\cos(x)}{x} = 0$$

Derivative of sin(x)

$$\frac{d}{dx}[\sin(x)] = \lim_{h\to 0} \frac{\sin(x+h)-\sin(x)}{h}$$

$$\frac{d}{dx}[\sin(x)] = \lim_{h\to 0} \frac{\sin(x)\cos(h)+\cos(x)\sin(h)-\sin(x)}{h}$$

$$\frac{d}{dx}[\sin(x)] = \lim_{h\to 0}(\frac{\sin(x)\cos(h)-\sin(x)}{h} + \frac{\cos(x)\sin(h)}{h})$$

$$\frac{d}{dx}[\sin(x)] = \lim_{h\to 0} \frac{\cos(x)\sin(h)}{h} + \lim_{h\to 0} -\frac{\sin(x)(1-\cos(h))}{h}$$

$$\frac{d}{dx}[\sin(x)] = \cos(x)\lim_{h\to 0} \frac{\sin(h)}{h} - \sin(x)\lim_{h\to 0} \frac{1-\cos(h)}{h}$$

$$\frac{d}{dx}[\sin(x)] = \cos(x)(1) - \sin(x)(0)$$

$$\frac{d}{dx}[\sin(x)] = \cos(x)$$

Derivative of cos(x)

$$\frac{d}{dx}[\cos(x)] = \lim_{h\to 0} \frac{\cos(x+h) - \cos(x)}{h}$$
$$\frac{d}{dx}[\cos(x)] = \lim_{h\to 0} \frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h}$$
$$\frac{d}{dx}[\cos(x)] = \lim_{h\to 0}\left(\frac{\cos(x)\cos(h) - \cos(x)}{h} - \frac{\sin(x)\sin(h)}{h}\right)$$
$$\frac{d}{dx}[\cos(x)] = \lim_{h\to 0} -\frac{\cos(x)(1 - \cos(h))}{h} - \lim_{h\to 0} \frac{\sin(x)\sin(h)}{h}$$
$$\frac{d}{dx}[\cos(x)] = -\cos(x)\lim_{h\to 0} \frac{1 - \cos(h)}{h} - \sin(x)\lim_{h\to 0} \frac{\sin(h)}{h}$$
$$\frac{d}{dx}[\cos(x)] = -\cos(x)(0) - \sin(x)(1)$$
$$\frac{d}{dx}[\cos(x)] = -\sin(x)$$

Derivative of e^x

$$\frac{d}{dx}[e^x] = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h}$$
$$\frac{d}{dx}[e^x] = \lim_{h\to 0} \frac{e^x e^h - e^x}{h}$$
$$\frac{d}{dx}[e^x] = e^x \lim_{h\to 0} \frac{e^h - 1}{h}$$
$$\frac{d}{dx}[e^x] = e^x \lim_{n\to 0} \frac{n}{\ln(n+1)}$$
$$\frac{d}{dx}[e^x] = e^x \lim_{n\to 0} \frac{1}{\frac{1}{n}\ln(n+1)}$$
$$\frac{d}{dx}[e^x] = e^x \lim_{n\to 0} \frac{1}{\ln(n+1)^{\frac{1}{n}}}$$

$$\frac{d}{dx}[e^x] = e^x \frac{1}{\ln(\lim_{n \to 0}(n+1)^{\frac{1}{n}})}$$

$$\frac{d}{dx}[e^x] = e^x \frac{1}{\ln(e)}$$

$$\frac{d}{dx}[e^x] = e^x \frac{1}{1}$$

$$\frac{d}{dx}[e^x] = e^x$$

Derivative of ln(x)

$$\frac{d}{dx}[\ln(x)] = \lim_{h \to 0} \frac{\ln(x+h) - \ln(x)}{h}$$

$$\frac{d}{dx}[\ln(x)] = \lim_{h \to 0} \frac{\ln(\frac{x+h}{x})}{h}$$

$$\frac{d}{dx}[\ln(x)] = \lim_{h \to 0} \frac{\ln(1 + \frac{h}{x})}{h}$$

$$\frac{d}{dx}[\ln(x)] = \lim_{h \to 0} \ln(1 + \frac{h}{x})^{\frac{1}{h}}$$

$$\frac{d}{dx}[\ln(x)] = \lim_{n \to 0} \ln(1 + n)^{\frac{1}{nx}}$$

$$\frac{d}{dx}[\ln(x)] = \frac{1}{x} \lim_{n \to 0} \ln(1 + n)^{\frac{1}{n}}$$

$$\frac{d}{dx}[\ln(x)] = \frac{1}{x} \ln(\lim_{n \to 0}(1 + n)^{\frac{1}{n}})$$

$$\frac{d}{dx}[\ln(x)] = \frac{1}{x} \ln(e)$$

$$\frac{d}{dx}[\ln(x)] = \frac{1}{x}$$

Derivative of a^x

$$\frac{d}{dx}[a^x] = \lim_{h\to 0} \frac{a^{x+h} - a^x}{h}$$
$$\frac{d}{dx}[a^x] = \lim_{h\to 0} \frac{a^x a^h - a^x}{h}$$
$$\frac{d}{dx}[a^x] = a^x \lim_{h\to 0} \frac{a^h - 1}{h}$$
$$\frac{d}{dx}[a^x] = a^x \lim_{h\to 0} \frac{e^{h\ln(a)} - 1}{h}$$
$$\frac{d}{dx}[a^x] = a^x \lim_{n\to 0} \frac{n}{\frac{\ln(n+1)}{\ln(a)}}$$
$$\frac{d}{dx}[a^x] = a^x \lim_{n\to 0} \frac{n\ln(a)}{\ln(n+1)}$$
$$\frac{d}{dx}[a^x] = \ln(a)a^x \lim_{n\to 0} \frac{n}{\ln(n+1)}$$
$$\frac{d}{dx}[a^x] = \ln(a)a^x$$

Derivative of $\log_a(x)$

$$\frac{d}{dx}[\log_a(x)] = \lim_{h\to 0} \frac{\log_a(x+h) - \log_a(x)}{h}$$
$$\frac{d}{dx}[\log_a(x)] = \lim_{h\to 0} \frac{\log_a(1+\frac{h}{x})}{h}$$

$$\frac{d}{dx}[\log_a(x)] = \lim_{h\to 0} \frac{\frac{\ln(1+\frac{h}{x})}{\ln(a)}}{h}$$
$$\frac{d}{dx}[\log_a(x)] = \lim_{h\to 0} \frac{\ln(1+\frac{h}{x})}{h\ln(a)}$$
$$\frac{d}{dx}[\log_a(x)] = \frac{1}{\ln(a)} \lim_{h\to 0} \frac{\ln(1+\frac{h}{x})}{h}$$
$$\frac{d}{dx}[\log_a(x)] = \frac{1}{\ln(a)}\frac{1}{x}$$
$$\frac{d}{dx}[\log_a(x)] = \frac{1}{\ln(a)x}$$

<u>If u(x) is Continuous at c, as Δx→0, Δu→0</u>

$$\lim_{x\to c} u(x) = u(c)$$
$$\lim_{x\to c}[u(x) - u(c)] = 0$$
$$\lim_{x\to c} x = c$$
$$\lim_{x\to c}[x - c] = 0$$
$$\Delta u = u(x) - u(c)$$
$$\Delta x = x - c$$
$$as\ \Delta x \to 0,\ x \to c,\ \Delta u \to 0$$

<u>Chain Rule</u>

$$\frac{d}{dx}[y(u(x))] = \frac{dy}{dx}$$

$$\frac{d}{dx}[y(u(x))] = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$
$$\frac{d}{dx}[y(u(x))] = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta u} \frac{\Delta u}{\Delta x}$$
$$\frac{d}{dx}[y(u(x))] = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta u} \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x}$$
$$\frac{d}{dx}[y(u(x))] = \lim_{\Delta u \to 0} \frac{\Delta y}{\Delta u} \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x}$$
$$\frac{d}{dx}[y(u(x))] = \frac{dy}{du} \frac{du}{dx}$$
$$\frac{d}{dx}[y(u(x))] = y'(u(x))u'(x)$$

Derivative of x^x

$$y(x) = x^x$$
$$\ln(y(x)) = x \ln(x)$$
$$\frac{d}{dx}[\ln(y(x))] = \frac{d}{dx}[x \ln(x)]$$
$$\frac{dy}{dx} \frac{1}{y(x)} = \ln(x) + 1$$
$$\frac{dy}{dx} = y(x)[\ln(x) + 1]$$
$$\frac{dy}{dx} = x^x(\ln(x) + 1)$$
$$\frac{d}{dx}[x^x] = x^x(\ln(x) + 1)$$

Derivative of arcsin(x)

$$y(x) = \sin^{-1}(x)$$
$$\sin(y(x)) = x$$
$$\frac{d}{dx}[\sin(y(x))] = \frac{d}{dx}[x]$$
$$y'(x)\cos(y(x)) = 1$$
$$\frac{d}{dx}[\sin^{-1}(x)] = \frac{1}{\cos(\sin^{-1}(x))}$$
$$\frac{d}{dx}[\sin^{-1}(x)] = \frac{1}{\sqrt{1-x^2}}$$

<u>Derivative of arccos(x)</u>

$$y(x) = \cos^{-1}(x)$$
$$\cos(y(x)) = x$$
$$\frac{d}{dx}[\cos(y(x))] = \frac{d}{dx}[x]$$
$$-y'(x)\sin(y(x)) = 1$$
$$\frac{d}{dx}[\cos^{-1}(x)] = -\frac{1}{\sin(\cos^{-1}(x))}$$
$$\frac{d}{dx}[\cos^{-1}(x)] = -\frac{1}{\sqrt{1-x^2}}$$

<u>Derivative of |x|</u>

$$\frac{d}{dx}[|x|] = \frac{d}{dx}[\sqrt{x^2}]$$
$$\frac{d}{dx}[|x|] = (2x)\left(\frac{1}{2\sqrt{x^2}}\right)$$

$$\frac{d}{dx}[|x|] = \frac{2x}{2\sqrt{x^2}}$$

$$\frac{d}{dx}[|x|] = \frac{x}{\sqrt{x^2}}$$

$$\frac{d}{dx}[|x|] = \frac{x}{|x|}$$